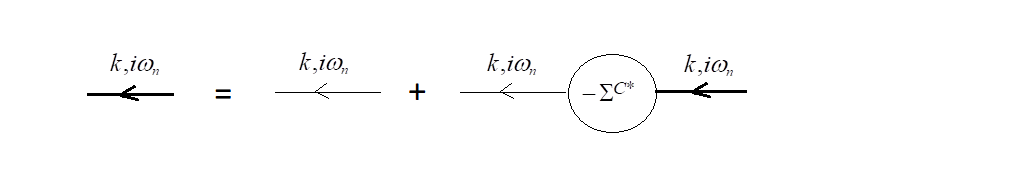
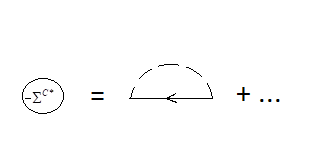
**Excitations**

**Electron Self-Energy**

In a metal, the motion of the ions doesn’t affect that of the electrons too much, especially because they do effectively screen the ions’ motion, reducing the ionic potential to a short ranged potential, which effectively makes the electron-ion interaction weak. But in a semi-conductor, say, it would be otherwise. Regardless, we can incorporate the effects of the lattice into the electron’s self energy via the usual expansion:



where,



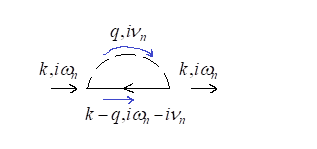
and,



Mahan considers that effective mass enhancement that the phonon-interactions impart to the electron. He remarks that it doesn’t affect the DC conductivity, contrary to expectations. Nor does it affect some other quantities. It does affect others though.

**Example**

Let’s just write out what the first term above is. So we’d have:



(Note we do not have a Hartree/tadpole/balloon diagram because that would have q = 0, which we eliminated from our Hamiltonian) And in symbols,



(the Ωq is implicitly that of the longitudinal branch – see Interaction file) And we could do the Matsubara sum with the contour integral approach, but I’ll not. Maybe I will. So using that identity (see Stat Mech/Math Appendix)



we can do the sum over frequencies, to get:



So we have:



Let’s note the following:



So can say,



So we finally come to:



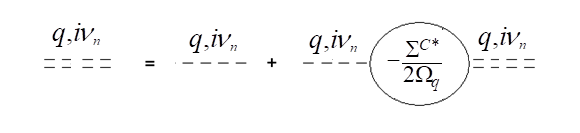
Could specialize to T = 0. Then nB(Ωq) = 0,



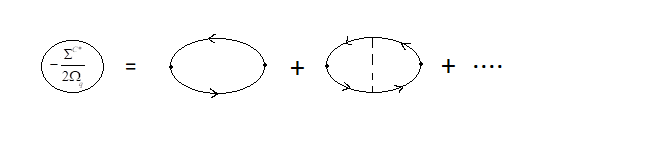
Hmmm. Think I’ll stop here.

**Phonon Self-Energy**

We can develop a self-energy expression for the phonon propagator too (this would just be in regards the longitudinal propagator – the transverse propagator has no self-energy in this interaction).



where



The recursion relation gives us,



which is:

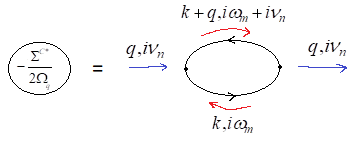


So now we have:



**Example**

Let’s consider the self-energy to first order.  This would be:



This is (the – sign on the RHS comes from the fermion loop):



I’ll note for future reference that the diagram w/o the g(q)2 is the RPA approximation to the e density-density correlation function, Πirr(RPA)C\*(q,iνn), i.e.,



(having done the spin sum – or equivalently put in factor of two vis a vis the fermion loop). So we can say,



Evaluating Π is easy in principle as it has just one isolated pole, and we can use the contour integration method to work it out.



And so then,



Let’s fill in our estimate for g(q)2 using the bare Coulomb interaction:



So then,



Borrowing some results from e-e folder, can say:



To get the new excitation spectrum, we want to solve the equation:



Let’s look at spectrum in small ω, small q limit. I think Ωq → ΩE so,



In Metals/Free Day/Equilibrium Properties/TF file there is formula for εRPA(q,ω) but I don’t want to bother. Well, in the small ω limit, we get the TF approximation:



But clearly this doesn’t give us an acoustic spectrum back when we solve for ω. In any event, I’ve read a few people saying that the electrons will quickly respond to a lattice ion’s change in position, and screen its potential, and thereby change the lattice’s long ranged plasma oscillations into short ranged acoustic oscillations so that ω(q→0) → 0, rather than ω(q→0) → ΩE. We can show this happens when we include e-e interactions (see e-e interactions + phonons folder). In that case, our Πirr above gets replaced by Π. Π includes e-e interactions whereas our Πirr does not, and that seems to make all the difference.